STRESS

Introduction

Any engineering Project, or component.

⟹ Determination of material and adequacy of physical dimensions.

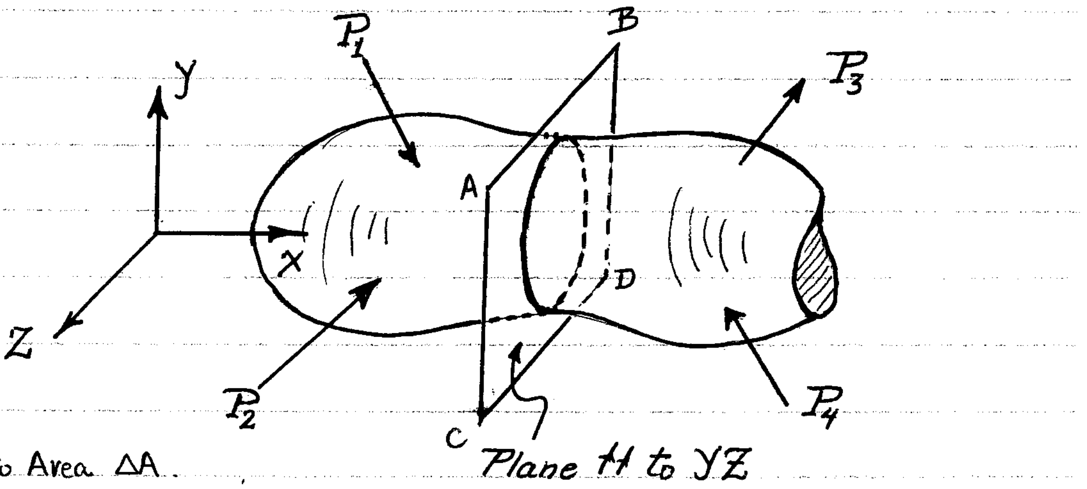
Examples : Walls of pressure vessels _ Floor of buildings _ Shafts of machines _ Wings of airplanes, etc...

Feasibility of Engineering Projects.

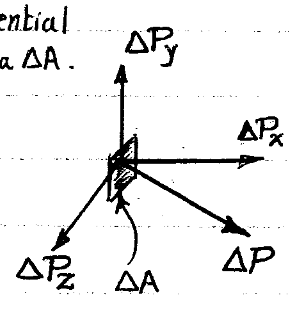
[1] Safety ⟶ Stability (maintaining a stable structure)
[2] Economy ⟶ Strength (providing adequate strength)
[3] Serviceability ⟶ Stiffness (" " stiffness)
(Clearances _ Deflections _ Spans).

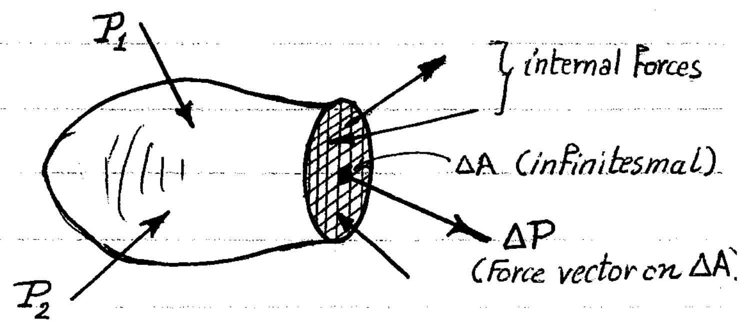
Concept

$$\{\Delta P\} = \begin{Bmatrix} \Delta P_x \\ \Delta P_y \\ \Delta P_z \end{Bmatrix}$$



- ΔP_x : Normal to Area ΔA.
- ΔP_y & ΔP_z : Tangential to Area ΔA.





$$\text{Stess} = \frac{\text{Forces}}{\text{Area}}.$$

Stresses
- Normal Stresses.
- Shearing Stresses

Normal Stresses

$$\tau_{xx} = \lim_{\Delta A \to 0} \frac{\Delta P_x}{\Delta A} = \sigma_x$$

Shearing Stresses

- $\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta P_y}{\Delta A}$ .... (Acting in a plane ⊥ to x and in y-dir.)
- $\tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta P_z}{\Delta A}$ ... (Acting in a plane ⊥ to x and in z-directi

Notation : τ_{ij}

Acting in a plane ⊥ to i-axis.

Direction of Stress in j-direction

Conversion of Units

	S.I units	USA units
FORCE	$1\,N \equiv 1\,\frac{kg.m}{sec^2}$.	Lb or kips
Stress	$1\,Pa = 1\,\frac{N}{m^2}$	psi or ksi (Lb/in^2) ($kips/in^2$)

$1\ kg \approx 2.2\ Lb$

$1\ ft \simeq 12\ in. \simeq 30.48\ cm$

$1\ Lb\text{-force} \simeq 4.4\ N$

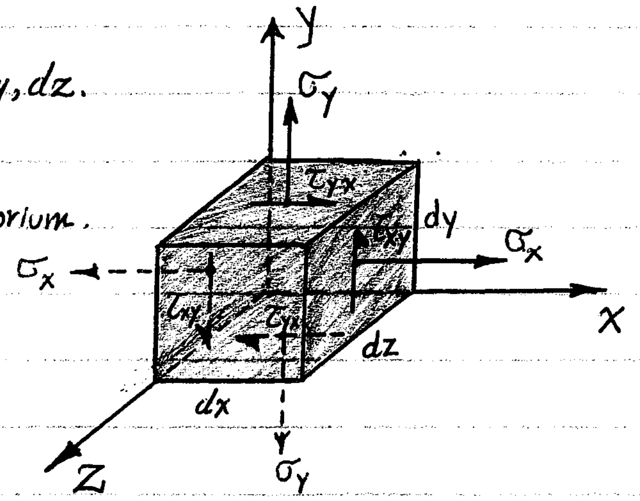
$1\ in. \simeq 2.54\ cm.$

$1\ psi \simeq 7000\ Pa \Rightarrow 1\ ksi \simeq 7\ MPa.$

$KPa = 10^3\ Pa$; $MPa = 10^6\ Pa$; $GPa = 10^9\ Pa.$

$Kip = 10^3\ Lb$

Two_Dimensional Stress _ Plane Stresses



Consider an infinitesmal cube , dx, dy, dz.

It can be verified that $\tau_{ij} = \tau_{ji}$.

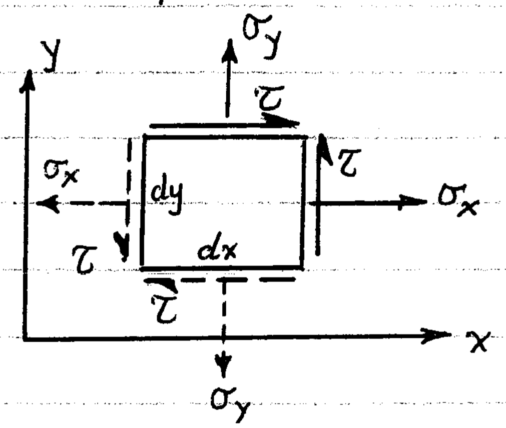
ex: By taking $\Sigma M/_{z\text{-axis}} = 0$ for equilibrium.

$\Rightarrow$ $\tau_{xy} = \tau_{yx}$.

Stresses in Plane X_Y are as shown .

then, let $\tau_{xy} = \tau_{yx} = \tau$.

Axial Problems



$\Rightarrow$ Biaxial Stresses : In the absence of all shear stresses and σ_3 ;(σ_z).

σ_1 or $\sigma_x = \checkmark$ $\qquad$ σ_3 or $\sigma_z = 0$

σ_2 or $\sigma_y = \checkmark$ $\qquad$ $\tau_{ij} \equiv \tau_{xy} = \tau_{xz} = \tau_{yz} = 0$.

ex: Thin sheets _ shell elements & fabrics.

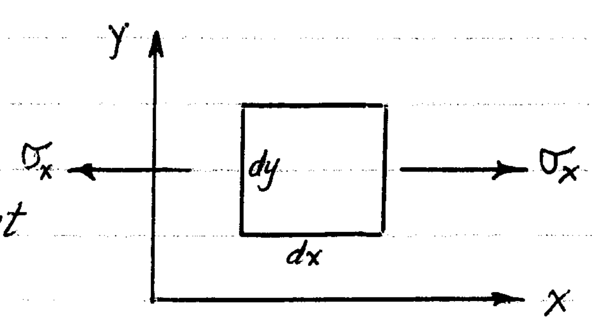
$\Rightarrow$ Triaxial Stresses : In the absence of all shear stresses .

$\sigma_1 = \checkmark$; $\sigma_2 = \checkmark$; $\sigma_3 = \checkmark$

$\tau_{ij} \equiv \tau_{xy} = \tau_{xz} = \tau_{yz} = 0$

ex: water body _ loose sand .

$\Rightarrow$ Uniaxial Stresses : In the absence of all shear stresses and two axial stresses .

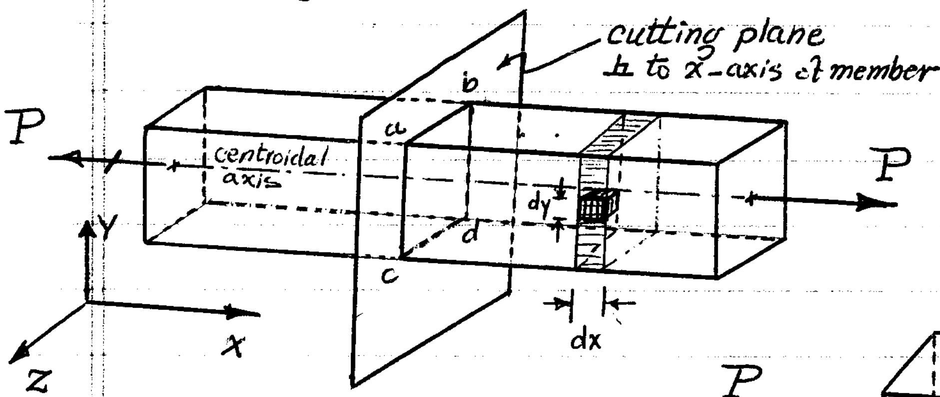


$\sigma_1 = \checkmark$; $\sigma_2 = \sigma_3 = 0$.

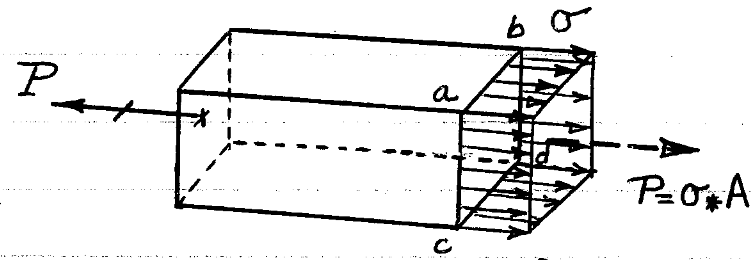
$\tau_{ij} \equiv \tau_{xy} = \tau_{xz} = \tau_{yz} = 0$

ex: axial members $\equiv$ Truss element

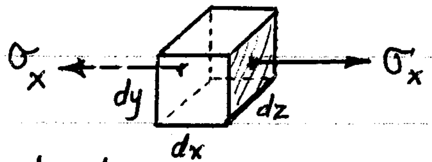
Axially Loaded Bars



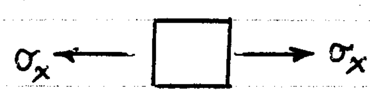
* Prismatic bar of cross-sectional area = A



uniformly distributed normal stress σ.



$$dP = \sigma_x * dy * dz$$
$$= \sigma_x * dA .$$



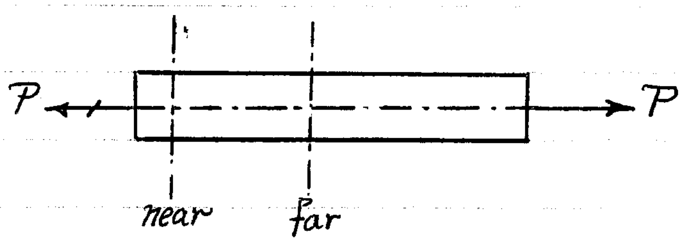
$$\Rightarrow \quad P = \int_A \sigma_x * dA = \sigma_x \int_A dA = \sigma_x . A .$$

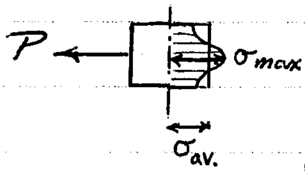
uniform stress

$$\sigma = \frac{\text{Force}}{\text{Area}} = \frac{P}{A} \quad .... \quad \left[\frac{N}{m^2}\right] \text{ or } \left[\frac{Lb}{in^2}\right]$$

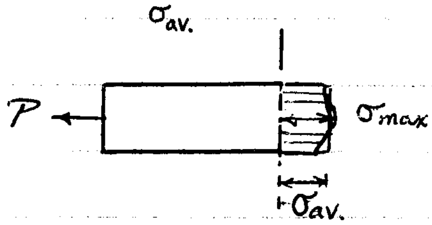
Stress Concentration

- $\sigma_{max} \rightarrow \infty$ under concentrated load (Theory).
- Equilibrium $\rightarrow P = \int_A \sigma\, dA$.

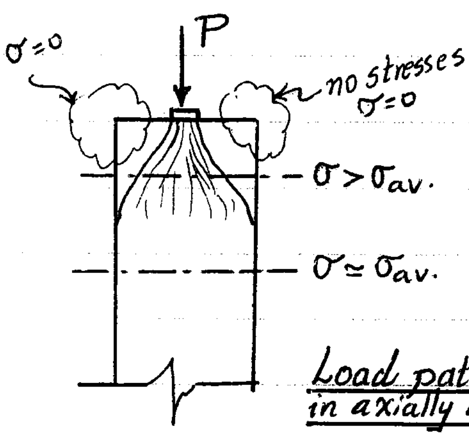




$\sigma_{max} > \sigma_{av.}$ ... (Stress Concentration

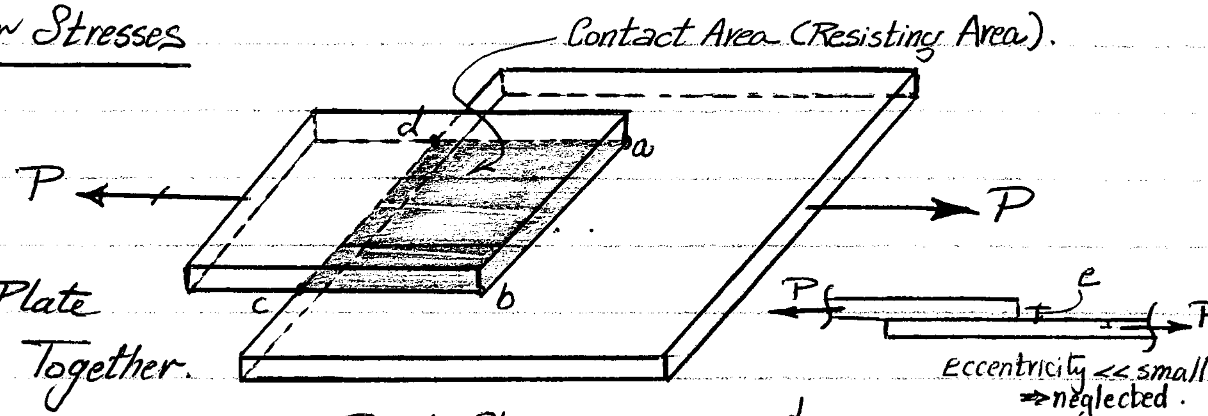


$\sigma_{max} \simeq \sigma_{av}$.



Load path in axially loaded column.

Shear Stresses



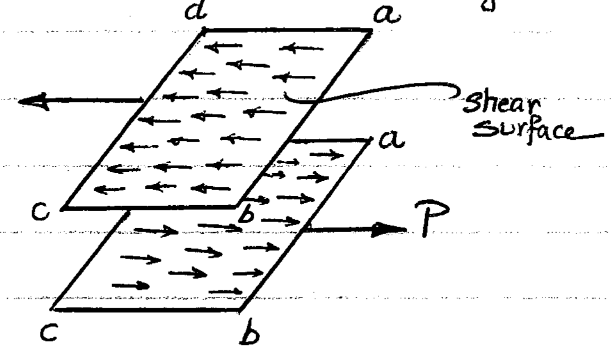
Two Plate Glued Together.

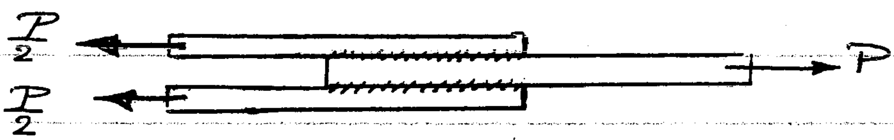
Single Shear

Shear force = P

Shear Area = Resisting Area = A_{abcd}.

Shear Stress = $\dfrac{P}{A_{a,b,c,d}} = \tau_{av.}$

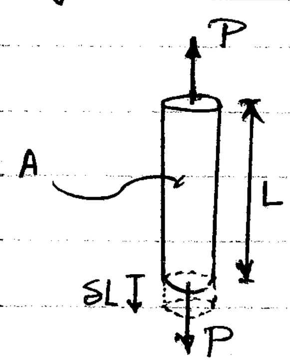




Double Shear $\Rightarrow \tau = \dfrac{P}{2A_{a,b,c,d}}$.

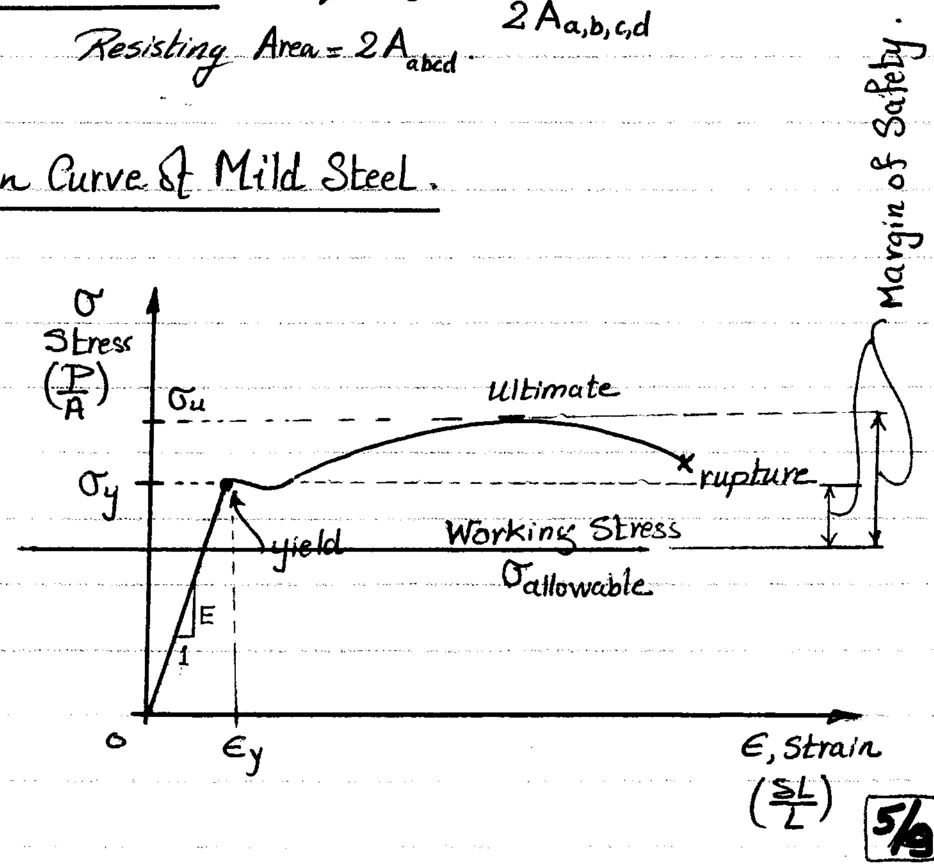
Resisting Area = $2A_{abcd}$.

❀ Typical Stress-Strain Curve of Mild Steel.



$\sigma = \dfrac{P}{A}$ ... Stress

$\epsilon = \dfrac{\delta L}{L}$ ... Strain



Design of Axially Loaded Members

- Factor of Safety (F.S).

$$F.S. = \frac{\text{Ultimate Load (Limit)}}{\text{Allowable Load}} = \frac{P_u}{P_{all.}}$$

OR $\longrightarrow$

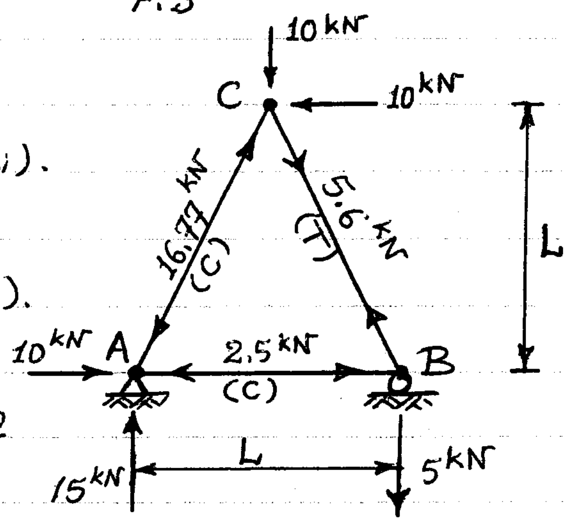
$$F.S = \frac{\text{Ultimate Stress (Limit)}}{\text{Allowable Stress}} = \frac{\sigma_u}{\sigma_{all}}.$$

- Why implementing safety factor in design?
  - Material defects.
  - Approximations in design.
  - Defects in Weld, connections, dimensions.
  - Effect of load repetition (Fatigue).
  - Uncertainty in Loads. (Live loads, wind & E.Q. loads).
  - Human error.

$$\Rightarrow \sigma_{all} = \frac{\sigma_u}{F.S}.$$

Example. (TRUSS PROBLEM).

Find Area of member BC



Given: $\sigma_u = 410{,}000$ kPa $\approxeq$ (60 ksi).

(Recall 1 ksi $\approx$ 7 MPa).

$\sigma_y = 250{,}000$ kPa $\cong$ (36 ksi).

$FS\,(\sigma_u) = 2.5$; $F.S = \frac{\sigma_u}{\sigma_{all}}$.

$\Rightarrow \sigma_u = 2.5 * \sigma_{all} \Rightarrow \sigma_{all} = \frac{410{,}000}{2.5}$

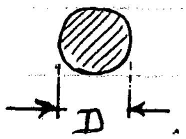
$\Rightarrow \sigma_{all} = 164{,}000$ kPa.

Member BC Force $= F_{BC} = 5.6$ kN ... (Tension).

$\sigma = \frac{F}{A} \Rightarrow A = \frac{F}{\sigma} \Rightarrow A_{BC} = \frac{F_{BC}}{\sigma_{BC}}$. Use $\sigma_{BC} = \sigma_{all}$.

$\Rightarrow A_{BC} = \frac{5.6 \text{ kN}}{164{,}000 \text{ kPa}} = 3.415 * 10^{-5}\ m^2 = 34.15\ mm^2$ .... (required area)

Using round bar of diameter D.



$$\Rightarrow A = \frac{\pi D^2}{4} \Rightarrow D_{req.} = \sqrt{\frac{4A_{req}}{\pi}} \approxeq \underline{\underline{6.6\ mm}}.$$

$\Rightarrow D_{provided} \geq D_{req.}$

$\Rightarrow \sigma \leq \sigma_{all}$ . .... (In order to satisfy strength requirement)

❀ In this context, it is worth mentioning that for design of compression members, it is not enough to check for strength, but members in comp need to satisfy stability against buckling.

- Compression Members
  - Strength requirement: $\sigma \leq \sigma_{all}$.
  - Stability requirement: $\sigma \leq \sigma_{cr}$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\text{Buckling Load}}{\text{Area}}$$

P_{cr} depends on: Length of member, Area of cross-section, Shape of cross-section, and connectivity of member.

<u>Member AB</u>

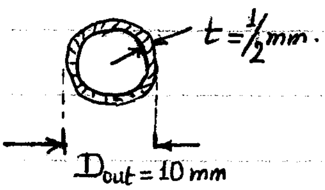
Force $= F_{AB} = 2.5\ kN$ .... (Compression). .... Assume No buckling

$$A_{AB} = \frac{F_{AB}}{\sigma_{all}} = \frac{2.5\ kN}{164{,}000\ kPa} = 1.524 * 10^{-5}\ m^2. = 15.24\ mm^2.$$

Round bar diameter, $D_{req.} = 4.41\ mm$.

Use $D_{prov.} = 4.5\ mm > D_{req}$ Ok.

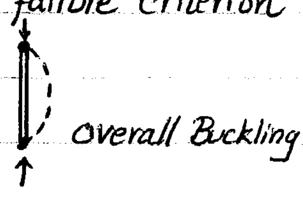
OR Try Tubular section, $D_{out} = 10\ mm$ and $t = \frac{1}{2}\ mm \Rightarrow D_{in.} = 9\ mm$.

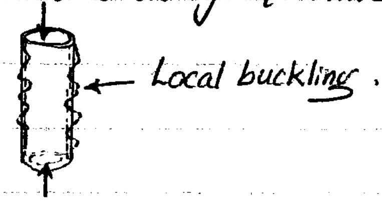


$$A_{provided} = \frac{\pi D_{out}^2}{4} - \frac{\pi D_{in}^2}{4} = 78.54\ mm^2 - 63.62\ mm^2$$

$$\approxeq 15\ mm^2 \quad \text{Ok}.$$

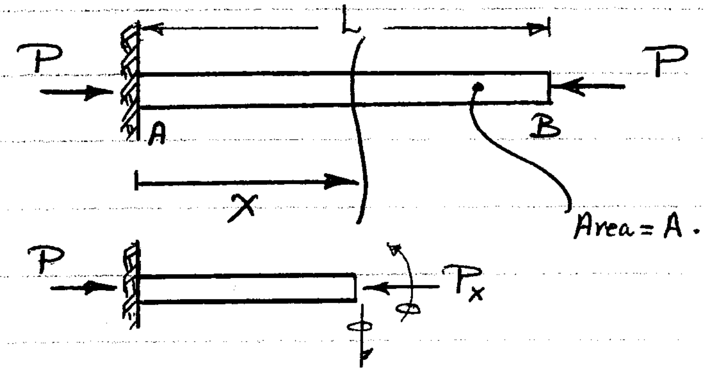
* Note that the second solution will provide larger inertia against buckling (overall buckling), however, one should check for another failure criterion that is local buckling of thin tubes.





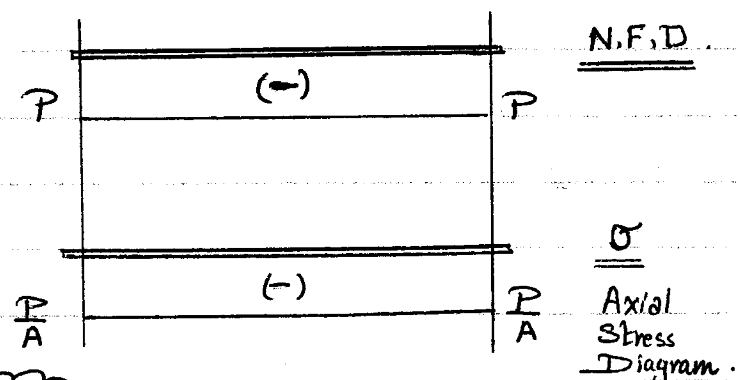
Axial Force and Axial Stress Diagrams

- $\Sigma X = 0$
  $\Rightarrow X_A = P \rightarrow$.
- $\Sigma Y = 0 \Rightarrow Y_A = 0$
- $\Sigma M_{/A} = 0 \Rightarrow M_A = 0$

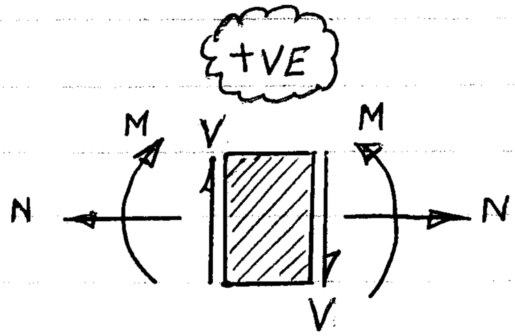


$\Sigma X = 0 \Rightarrow P_x = P \quad 0 < x < L.$

* In axially loaded members: $N = \checkmark$
$V = 0$
$M = 0$
at any section.



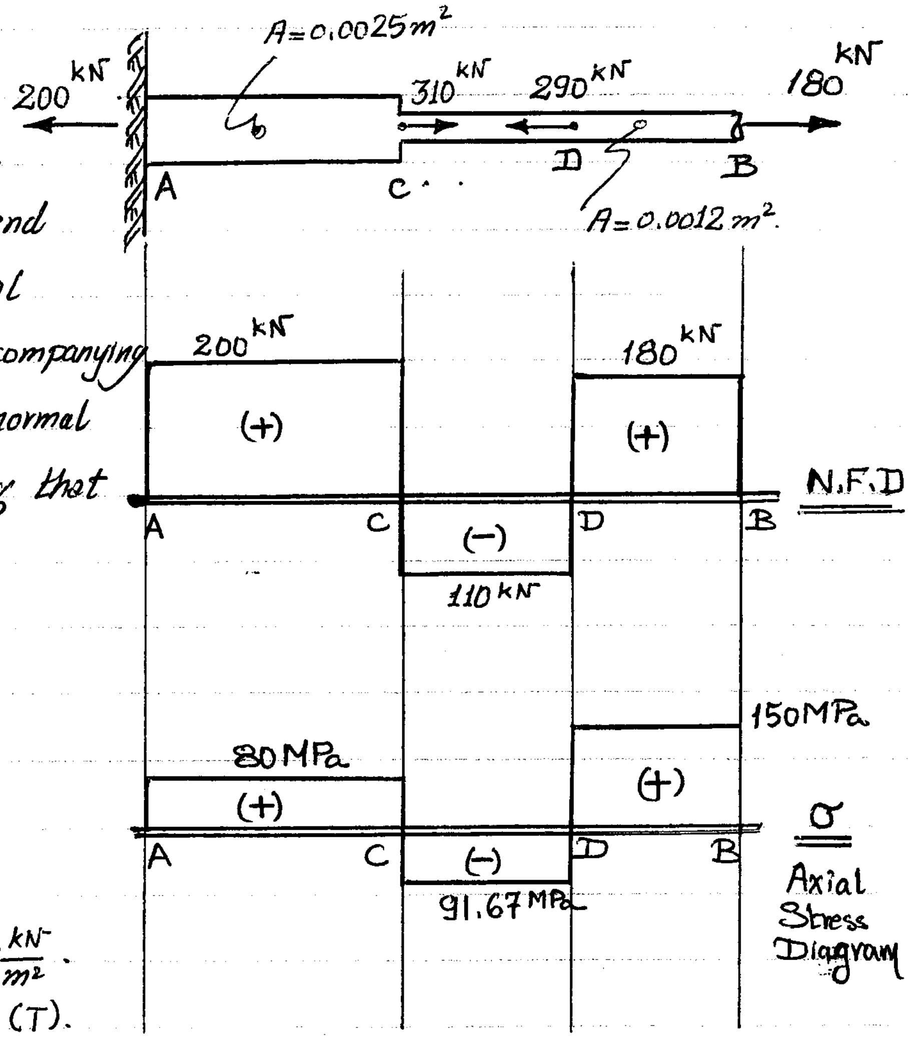
Recall,



SIGN CONVENTION
FOR INTERNAL FORCES

Example

A rod of variable cross-section built in at one end is subjected to three axial forces as shown in the accompanying figure. Find the max. normal stress and F.S. knowing that $\sigma_{Limit} = 290$ MPa.



Reactions

$\Sigma X = 0$

$\Rightarrow X_A = 200\ kN \leftarrow$

- $\sigma_{DB} = \dfrac{180\ kN}{0.0012\ m^2} = 150{,}000\ \dfrac{kN}{m^2}$
  $= 150$ MPa (T).

- $\sigma_{CD} = \dfrac{-110\ kN}{0.0012\ m^2} = -91{,}667\ \dfrac{kN}{m^2}$
  $= 91.667$ MPa (C).

- $\sigma_{AC} = \dfrac{200\ kN}{0.0025\ m^2} = 80{,}000\ \dfrac{kN}{m^2}$
  $= 80$ MPa (T).

$\Rightarrow \underline{\sigma_{max.} = 150 \text{ MPa}}$.
(SEGMENT BD)

Notice $\sigma_{min.} = 80$ MPa ...(SEGMENT AC).

* Provided Factor of Safety (F.S) $= \dfrac{\sigma_{Limit}}{\sigma_{actual}}$

- Segment BD $\Rightarrow$ F.S $= \dfrac{290}{150} = 1.933 \longrightarrow$ Minimus F.S.
- Segment CD $\Rightarrow$ F.S $= \dfrac{290}{91.67} = 3.164$.
- Segment AC $\Rightarrow$ F.S $= \dfrac{290}{80} = 3.625$.